Comparison of stochastic reduced order model and stochastic collocation methods for the sensitivity analysis of crosstalk

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Abstract

Sensitivity analysis is a crucial tool to reduce the randomness dimension of stochastic problems, thus to reduce computation cost. However, the efficacy of different stochastic methods for sensitivity analysis was totally missing. Therefore, two popular stochastic methods, i.e., the stochastic reduced order model (SROM) and stochastic collocation (SC) methods, are performed to analyse crosstalk sensitivity to different cable variables. The Monte-Carlo (MC) method is used as the reference. Both the SROM and SC methods are shown to be much more efficient by orders of magnitude compared with the MC method. The result also shows that the SROM performance is not as remarkable as that of the SC method in terms of efficiency. As a result, the SC method could be recommended as a promising efficient approach for sensitivity analysis in stochastic electromagnetic compatibility (EMC) problems.

1 Introduction

The susceptibility of a cable to crossstalk is an important aspect of electromagnetic compatibility (EMC) performance. Predicting the crosstalk level at the early stage is crucial to ensure a satisfactory performance of the cable in the real working environment.

Traditional deterministic prediction of crosstalk may be unconvincing in the real scenario as the stochastic nature of cable variables is not properly considered. Therefore, statistical methods [1]-[3] should be used to capture the variation of crosstalk due to parametric uncertainty.

The computational cost of statistical analysis is related to the randomness dimension of the problem, i.e., the number of random input variables. For large randomness dimension, the computational cost of the analysis may become prohibitively expensive. Therefore, it is desirable to reduce the randomness dimension, thus to ease the statistical analysis.

To this end, one can identify random variables with weak influences on the variability of the output, and ignore the uncertainty of the weak variable during the statistical analysis. As a result, much less computational cost would be needed without sacrificing accuracy.

Weak random variables can be identified using sensitivity analysis [4]. In each sensitivity analysis, the uncertainty degree of the random variable under investigation is solely propagated to the system response. The isolated impact of the variable on the output variability can be judged by comparing their uncertainty degrees.

Clearly, statistical approaches are needed to implement sensitivity analysis. It is desirable to conduct sensitivity analysis as efficiently as possible. The stochastic reduced order model (SROM) method was newly proposed in [3] as a potential alternative to the stochastic collocation (SC) [2] method. Both methods are nonintrusive and more efficient than the Monte-Carlo (MC) method [1]. However, the relative goodness of SROM over SC or vice versa for sensitivity analysis was unknown. Therefore, both the SROM and SC methods are implemented to analyse the crosstalk sensitivity to different cable variables. Their performances are compared regarding the efficiency of performing sensitivity analysis. Also, the stochastic nature of the cable configuration in this paper is more representative of real scenarios, compared to the preliminary sensitivity analysis of crosstalk in [5]. Detailed discussion can be found in Section 4.1.

The following sections are organised as follows: Section 2 presents brief introductions of the SROM and SC methods. The cable structure as a three-conductor transmission line is described in Section 3. In Section 4, the efficiency of the SROM and SC methods for crosstalk sensitivity analysis is thoroughly discussed. Finally, the conclusion is presented in Section 5.

2 SROM and SC

The brief introductions of the SROM and SC methods are presented in this section.

2.1 SROM method

The SROM method offers an innovative idea to perform statistical analysis efficiently. Let us start by defining $\boldsymbol{X} = [X_1, X_2, \ldots, X_D]$ as a D-dimensional variable to represent the input space. Each dimension X_i ($1 \le i \le D$) of $\boldsymbol{X}$ describes the variation of a random variable.

A SROM model $\tilde{\boldsymbol{X}}$ is a set of selected samples with assigned probabilities, in order to give accurate representation of the statistics of the random variable $\boldsymbol{X}$. Once the SROM model $\tilde{\boldsymbol{X}}$ is constructed, the SROM-based output model $\tilde{\boldsymbol{Y}}$ for the system response $\boldsymbol{Y}$ can be obtained using the deterministic solver. The statistics of the SROM-based output $\tilde{\boldsymbol{Y}}$ can be produced to approximate the statistical properties of the actual response $\boldsymbol{Y}$.

Clearly, the computational cost of the SROM method is dependent on the number of samples in the SROM model $\tilde{\boldsymbol{X}}$. For the detailed introduction of the SROM method, please see [6].

2.2 SC method

The SC method is an efficient statistical approach, especially for high randomness dimension. The aim of the SC method is to derive the analytical relationship between random input variables and the output response.

To derive this analytical formula, two steps need to be done. The first step is to select collocation points using sparse grid sampling computed via the Smolyak algorithm [2], [7]. The number of collocation points is determined by the randomness dimension D and the construction level k of the Smolyak algorithm. For a fixed value of D, the increment of k increases the number of collocation points as well as accuracy. In the second step, the system output values at collocation points are calculated by calling the deterministic solver. The analytical relationship can then be derived by performing Lagrange interpolation between those output values.

Taking this analytical formula as the replacement of the deterministic solver, the statistics of the output response can be obtained numerically or analytically with ease. The computational cost of the SC method is mainly dependent on the number of collocation points. Detailed description of the SC method can be found in [2] and [7].

3 Cable model

This section introduces the configuration of the three-conductor transmission line as the cable model. As shown in Fig. 1, the line connected to the voltage source V_S is the generator wire. The other line is the receptor wire which the crosstalk is induced to. The two lines have the same length L and are straight parallel above the ground plane. The ground plane is used to form loops for the generator and receptor circuits.

3.1 Types of random variables

The uncertainty in the cable configuration is typically from the following sources: the termination load of each circuit, the height of each line above the ground plane, and the radius of each conductor. For the three-conductor transmission line in

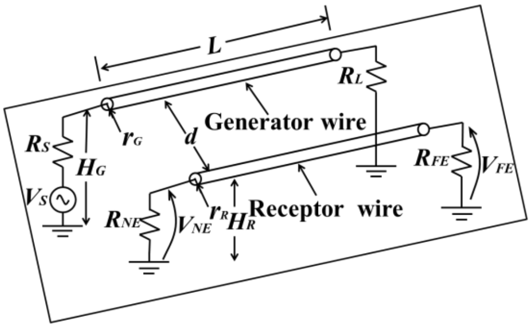

Fig. 1. Configuration of three-conductor transmission lines [5].

Fig. 1, the termination loads in the generator circuit are the source impedance R_S and load impedance R_L. In the receptor circuit, the receptor wire is terminated to the ground plane via the *near-end* load R_{NE} and *far-end* load R_{FE}. The heights of the two lines are expressed by H_G and H_R (with the footnote G/R denoting the generator/receptor wire). The radii of the two conductors are indicated by r_G and r_R, and the distance between the two lines is denoted using d.

3.2 System response

Interference is imposed on the receptor circuit when the voltage source V_S is switched on. The term *near-end* crosstalk (*NEXT*) is used to quantify the interfering effect and defined as follows:

$$NEXT = \frac{V_{NE}}{V_S} \tag{1}$$

where V_{NE} indicates the coupled voltage across the *near-end* load R_{NE} in the receptor circuit.

3.3 Solution of crosstalk *NEXT*

Having known the value of each variable in Section 3.1, the value of *NEXT* can be produced using the deterministic solver in [8].

4 Sensitivity analysis using SROM and SC

The sensitivity of crosstalk to different uncertainty sources in a three-conductor transmission line is evaluated using the SROM and SC methods.

Random variables are assumed to follow Gaussian distributions with statistics given in Table 1. The uncertainty degree of each variable is described using the coefficient of

Input Variable	μ	σ	*COV*
H_G (mm)	10	1	0.1
H_R (mm)	10	1	0.1
r_G (mm)	0.4064	0.0406	0.1
r_R (mm)	0.4064	0.0406	0.1
R_S (Ω)	50	5	0.1
R_L (Ω)	50	5	0.1
R_{NE} (Ω)	50	5	0.1
R_{FE} (Ω)	50	5	0.1

Table 1: Statistical properties of random input variables.

variance (*COV*) [4] defined as the ratio of the standard deviation σ to the mean value μ. A large value of *COV* indicates a wide variability of the random variable. In each sensitivity analysis, the investigated random variable is assumed to have $COV = 0.1$.

By default, the simulation is performed at the frequency f = 400 MHz, transmission line length L = 8 m and wire separation d = 8 mm. The SROM and SC results are benchmarked against the reference result given by 1,000,000 MC simulations.

4.1 Random heights of two lines

In this section, the uncertain sources are the height H_G of the generator wire and height H_R of the receptor wire with the statistical property given in Table 1. The sensitivity analysis of crosstalk to cable height was first presented in [5] based on a simplified assumption. Specifically, the assumption in [5] introduced a 1-D random variable H as $H = H_G = H_R$, and the sensitivity analysis was performed with respect to H. This assumption implied that the variables H_G and H_R would take identical values in a simulation. For example, we may have H_G = 9.5 mm, H_R = 9.5 mm and H_G = 10.3 mm, H_R = 10.3 mm in two simulations. However, in practice, the values of H_G and H_R are not necessarily the same. For example, we may encounter H_G = 9.5 mm and H_R = 10.3 mm in a simulation.

The simplified assumption in [5] is rectified in this study by regarding H_G and H_R as two independent random variables to accurately represent the stochastic nature. Let $\boldsymbol{X} = [H_G, H_R]$ be a 2-D random variable. The SROM-based input $\tilde{\boldsymbol{X}}$ can be constructed and used to derive the statistics of crosstalk *NEXT*.

To implement the SC method at randomness dimension D = 2, the number of collocation points are 5, 13, 29, 65, 145, 321, and 705, for $k = 1, 2, \ldots, 7$, respectively. The number of samples in the SROM-based input $\tilde{\boldsymbol{X}}$ is chosen the same as the SC collocation point number for fair comparison.

Fig. 2 shows the performance of SROM and SC to produce accurate value of *COV* (*NEXT*). As can be seen, the relative goodness of SC over SROM is obvious in this case. Specifically, the SC method only needs 13 samples to converge within the error of 0.7% with respect to the reference *COV* (*NEXT*), whereas as the SROM method requires 145 samples. Despite this, the SROM method is still much more efficient compared with the MC method, as at least 7000 MC samples are needed to converge to the reference result. Therefore, the sensitivity analysis using MC can be expedited by 7000/145 ≈ 48 times using SROM or by 7000/13 ≈ 538 times using SC. Please note that Fig. 2 only shows one possible MC performance. This is because the MC performance with the computational cost too small for convergence could vary. This phenomenon is also true for the MC performance shown later in Section 4.2 and Section 4.3.

Output Response	μ	σ	*COV*
NEXT	0.0352	0.0010	0.0288

Table 2: Theoretical statistics of *NEXT* when the wire heights are uncertain with $COV = 0.1$.

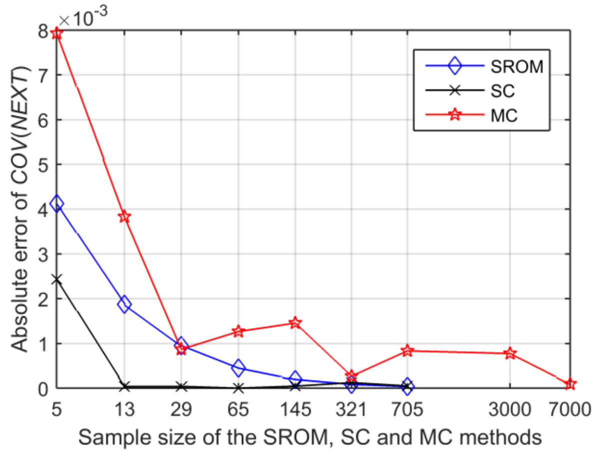

Fig. 2. Convergence rates of the SROM, SC, and MC methods to produce accurate *COV* (*NEXT*), when only the wire heights H_G and H_R are the random parametric inputs.

Table 2 shows that wire heights are very weak variables to influence the variability of crosstalk *NEXT*. This is because the uncertainty degree of *NEXT* is only 0.0288, much smaller than the uncertainty degree $COV = 0.1$ of the wire. This means that the uncertainty is significantly reduced in the propagation process. As a result, the uncertainty of the wire height may be ignored to reduce the randomness dimension of the statistical analysis of crosstalk.

Output Response	μ	σ	*COV*
NEXT	0.0352	0.0015	0.0423

Table 3: Theoretical statistics of *NEXT* when the wire radii are uncertain with $COV = 0.1$.

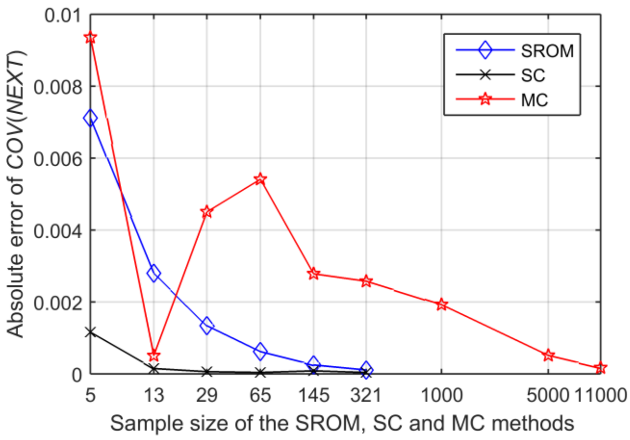

Fig. 3. Convergence rates of the SROM, SC, and MC methods to produce accurate *COV* (*NEXT*), when only the wire radii r_G and r_R are the random parametric inputs.

4.2 Random radii of two lines

This section presents the sensitivity analysis of crosstalk to the radii r_G and r_R of two wires. Similar to the contribution of Section 4.1, the simplified assumption of introducing a 1-D variable r with $r = r_G = r_R$ in [5] is now rectified by treating r_G and r_R as independent variables. To account for the independence, sensitivity analysis is performed with respect to the 2-D random variable $\boldsymbol{X} = [r_G, r_R]$.

As shown in Fig. 3, the SC method is more efficient than the SROM method for this analysis. Specifically, the SROM method needs 145 samples to converge within the error of 0.6%, compared to only 13 samples required by the SC method. On the other hand, at least 11,000 samples are needed by the MC method to converge to the same accuracy. Therefore, the MC computational cost can be reduced by a factor of 11000/145 ≈ 76 using SROM or a factor of 11000/13 ≈ 846 using SC.

As shown in Table 3, the uncertainty degree of crosstalk *NEXT* is *COV* (*NEXT*) = 0.0423, as a response to the variability of radii r_G and r_R with COV = 0.1. Clearly, the radii of wires can be regarded as weak variables. Please note that compared with the wire height, the influence of the wire radius on crosstalk variability is stronger.

4.3 Random impedance loads of two lines

In this section, all the impedance loads (i.e., R_S, R_L, R_{NE}, and R_{FE}) are assumed to be random variables with statistics given in Table 1. The sensitivity analysis of crosstalk *NEXT* is performed with respect to these four variables. Again, unlike the assumption of introducing a 1-D variable T as $T = R_S = R_L = R_{NE} = R_{FE}$ in [5], the four random impedance loads are assumed to be independent of each other in this study. To account for these four variables, a 4-D variable $\boldsymbol{X} = [R_S, R_L, R_{NE}, R_{FE}]$ is introduced. The uncertainty degree of crosstalk *NEXT* is quantified using the SROM and SC methods. The performances of SROM and SC are compared at the sample sizes of 9, 41, 137, 401, 1105, and 2929, which are the number of SC collocation points with k = 1, 2, …, 6, respectively, at D = 4.

As shown in Fig. 4, the SC method has a faster convergence rate compared to the SROM method. Specifically, SROM needs 401 samples to produce the converged result within the error of 1.1%, whereas SC only requires 137 samples. In this case, at least 14,000 samples are needed by MC to converge to the same accuracy. Therefore, the sensitivity analysis with MC can be accelerated by a factor of 14000/401 ≈ 35 using SROM or a factor of 14000/137 ≈ 102 using SC.

As shown in Table 4, the uncertainty degree of crosstalk *NEXT* as a response to random impedance loads is quantified to be *COV* (*NEXT*) = 0.0957. Compared to the uncertainty degrees of crosstalk caused by the wire height and radius, the termination load is found to be the strongest variable to impact crosstalk variability. From the results of three sensitivity analysis above, it is interesting to note that SC is more efficient than SROM. This may be due to the fact that the crosstalk is typically in linear relationship with the investigated variable in this paper. The SC interpolation to recover a linear relationship is usually very efficient.

Output Response	μ	σ	*COV*
NEXT	0.0352	0.0034	0.0957

Table 4: Theoretical statistics of *NEXT* when impedance loads are uncertain with *COV* = 0.1.

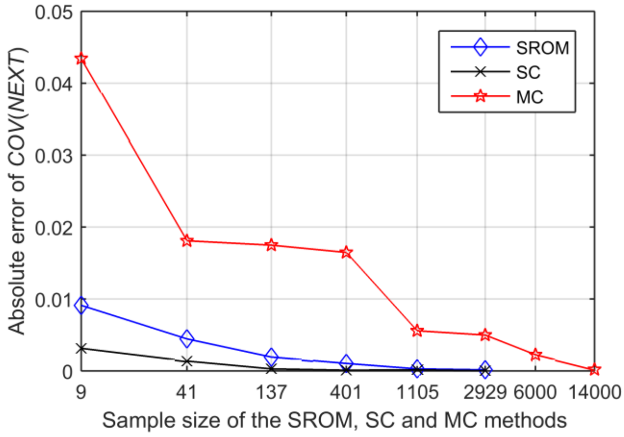

Fig. 4. Convergence rates of the SROM, SC, and MC methods to produce accurate *COV* (*NEXT*), when only the impedance loads R_S, R_L, R_{NE}, and R_{FE} are the random parametric inputs.

Example		$\boldsymbol{X}=[H_G, H_R]$	$\boldsymbol{X}=[r_G, r_R]$	$\boldsymbol{X}=[R_S, R_L, R_{NE}, R_{FE}]$
SROM	Time (s)	8.6	7.8	15.1
	Samples	145	145	401
SC	Time (s)	3.2	3.5	5.2
	Samples	13	13	137
MC	Time (s)	87.4	129.5	165.2
	Samples	7000	11000	14000

Table 5: Comparison of the SROM, SC, and MC methods.

The number of required samples and implementation time of each method to converge to good accuracy in each case are summarised in Table 5, by using a CPU of 3.2 GHz and RAM of 24 GB.

5 Conclusion

The sensitivity analysis of crosstalk to different cable variables has been performed based on accurate description of stochastic parametric nature. The result of the sensitivity analysis has revealed the impact of each variable on crosstalk variability.

The efficiency of SROM and SC to implement sensitivity analysis has been compared. Both SROM and SC are more efficient than MC by orders of magnitude for sensitivity analysis of crosstalk. Also, the SROM method is found to be less efficient compared to the SC method. Therefore, the SC method could be recommended as an efficient tool to perform sensitivity/statistical analysis. However, this study may not be sufficient to conclude that the SC method generally outperforms the SROM method in terms of efficiency. Further applications of SROM and SC in other EMC problems shall be needed to produce a comprehensive evaluation.

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