​**Probabilities and preschoolers: do tangible vs virtual manipulatives, sample space and repetition matter?**

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Probabilities tend to become an integral part of early childhood mathematics curricula. Research has shown that at the age of 4, children indicate basics of probabilistic reasoning and can engage with probabilistic tasks and uncertainty. The aim of this study is to examine whether methodological and design alterations influence children’s inferences of the most probable event. Children (N = 480), aged 4–6 years, participated in choice tasks involving probabilistic predictions after repeated random draws, in groups of 3. Participation was either through the interaction with tangible manipulatives (Condition 1) or through computer-based manipulatives (Condition 2). During participation, children recorded their initial predictions and the actual outcomes on specially designed sheets and each task was repeated three times. Findings imply that preschoolers appreciate what is more probable and show a significantly better understanding of the likelihood of events when interacting with tangible rather than computer-based manipulatives and when the sample space is simpler (p < .01); thus, repetition was not found to be significant. Such results are important when designing and embedding basic probabilistic notions in the early childhood classroom, aiming at promoting children’s probability literacy.

1. **Introduction**

Learning about the world requires learning probabilistic relationships (Yurovsky et al 2013). From early in life, children experience and interact with the world around them while understanding what is possible, probable, certain, random or risky. They make inferences and predictions, try out various choices and options whilst developing causal and statistical reasoning (Gopnik and Schulz 2007). Understanding rules in which cues do or do not predict outcomes, considering alternative pathways, calculating the odds, making causal connections, taking informed decisions based on judgment, are processes that emerge from early in life. Young infants have been found able to make predictions from probabilistic events in their environments in a number of tasks (i.e. Bulf, Johson and Valenza 2011; Tenenbaum et al, 2011). Similarly, young children have been found to express data and probability sense especially through intuitive accounts, before they enter formal education (i.e. Girotto et al 2016; Nikiforidou, Pange, Chadjipadelis 2013). On many occasions, children work out the probabilities of particular outcomes and through experience, intuition and cognitive processing they develop an understanding of the possible gains and losses in different situations; how possible is it for me to fall if I climb that tree? If I choose this game then who will possibly join me? What if I mixed yellow and red paint?

From an educational perspective, over the last decades there has been an increased interest in embedding basic notions of probabilities and statistics in early childhood mathematical curricula. For example, in the Western Australian Curriculum: Mathematics is organised around the interaction of three content strands including *Statistics and Probability* from pre-primary education (Government of Western Australia 2016); the Australian Curriculum: Mathematics (ACARA 2015) includes in Foundation Year, *data representation* and *interpretation* as a content descriptor; the Common Core State Standards for Mathematics (National Council of Teachers of Mathematics, 2012) introduce statistical notions through the *Measurement and Data* strand; in the he Korean Mathematics Curriculum (Ministry of Education Science and Technology, 2011) *Probability and Statistics* are introduced in grade 1. These initiatives highlight the significance of exploring probabilities from early ages and call for further research and examination in identifying the best possible methodological and pedagogical approaches.

Early math experiences, including statistics and probabilities, have been found to be strongly correlated to later educational success and long-term thriving. There have been studies showing the connection between children’s early math experiences and later mathematics achievement in school and beyond (Claessens & Engel, 2013; Hannula-Sormunen, Lehtinen & Rasanen, 2015). However, Watts, Duncan, Clements and Sarama (2018) question this strong assumption by arguing that the skill-building process from preschool to primary is not monotonous and depends on the subsequent learning environments. In simple words, they propose that early mathematical knowledge can only be paired with later mathematical knowledge if there is correct mix of content and teaching. This reflects the notion of the ‘spiral curriculum’ (Bruner, 1960), where: ‘*if the understanding of number, measure, and probability is judged crucial in the pursuit of science, then instruction in these subjects should begin as intellectually honestly and as early as possible in a manner consistent with the child's forms of thought. Let the topics be developed and redeveloped in later grades*’ (pp.53-54). Thus, Bruner (1960) recognises the importance of introducing number, measure and probability as early as possible, in a way that relates to the child’s cognition and understanding.

In enhancing probability literacy (Gal, 2012) in early childhood, like with older ages, the pedagogy and methodology play key role. Specifically, task design and complexity (Schlottmann and Wilkening 2012; Kinnear and Clark 2014) influence how probabilistic intuition and analysis can interplay in young ages. The aim of this study is to investigate how young children respond to probabilistic tasks through methodological variations. These variations refer to changes in the distribution of the sample space, to the use of tangible vs computer-based manipulatives and to the impact of repetition of the task. Do these 3 features influence children’s engagement in an animal-card guessing game? Does the manipulative (tangible vs computer-based) make a difference on children’s predictive choices? Do changes in the distribution of the sample space influence children’s inferences? Moreover, a learning trend is hypothesised as children participated in the same task for three consecutive times.

***1.1 Probabilistic thinking in early childhood and probability literacy***

Piaget and Inhelder (1975) investigated the notion of chance in young children systematically. They found that children at the pre-operational stage, between 4 and 7 years, are not able to differentiate certainty (non-randomness) and uncertainty (randomness), because they lack advanced logical and arithmetical operations. Under the Piagetian approach, children at this stage have limited cognitive capacity to understand irreversibility, deduction, random mixing and random distribution. Instead they base their probabilistic judgments on phenomenism, in the sense that they accept as real what they see happening and on egocentrism, where subjective and intrinsic accounts dominate children’s thinking. Piaget and Inhelder (1975) found that through random drawings, children after 7 or 8 years, develop schemas of combining possible trajectories.

Nevertheless, even though children have not yet developed advanced intellectual mechanisms, they face probabilistic events and respond to them on a daily basis, where guessing, prediction, choice and decision are required. This happens through intuitions, according to Fischbein (1975), who underlines the differentiation between the concept of probability and the intuition of probability. Intuitions are forms of immediate cognition, intrinsic to reasoning, stabilised by experience (Fischbein 1975). The concept of probability is explicit and grounded on computations, whereas the intuition of probability is subjective and ‘integral part of intelligent behaviour’ (p.117). Under this perspective, preschool children can demonstrate superior ability to estimate odds, to distinguish the un-predictable and to express intuitive preoperational inferences.

Recent studies have shown that children as young as 5 demonstrate an understanding of probabilities and expected value, adjust preferences based upon probabilities and possess specific concepts and skills associated with probabilistic reasoning (Batanero et al 2016; Jones et al 1999; Nikiforidou and Pange 2010). Preschoolers and infants can make use of random sampling and show sensitivity for probabilities (Denison and Xu 2014), they can understand the probability of an event in binary choices (Falk, Yudilevich-Assouline, and Elstein 2012) and they can make use of probabilistic evidence in order to infer about causal strength (Kushnir and Gopnik 2007). Fisk, Bury and Holden (2006) found that children aged 4-5 would commit the conjunction fallacy while participating in tasks involving choice between the more likely of two events, a single event and a joint event (conjunctive or disjunctive). Boyer (2007) used a computerised sequential event sampling decision-making task to identify that 5 and 6 year olds select the more probable outcome by demonstrating intuitive sensitivity to probability. Similarly, Girotto et al. (2016) found that in probabilistic choice tasks 5 year olds made optimal choices, whereas 3-4 year olds based their responses on randomness and/or superficial heuristics and Denison et al (2013) found that 4 and 5 year olds indicated sophisticated probabilistic inferences in a series of 4 experiments.

Probability literacy can be defined as “*the ability to access, use, interpret, and communicate probability-related information and ideas, in order to engage and effectively manage the demands of real-world roles and tasks involving uncertainty and risk*” (Gal 2012, 4). Gal (2005) supports that probability literacy unpicks knowledge and dispositions regarding probabilistic matters. It is about the exploration of big ideas, like, variation, randomness, predictability/uncertainty, as well as having critical stance, beliefs and attitudes and personal sentiments regarding uncertainty. He concludes that probability literacy is not about emphasising the technical or procedural knowledge only but has to reflect the dispositional side too, in a coordinated and balanced way. In the same direction, Borovcnik (2016) views probabilistic literacy as *‘the ability to use relevant concepts and methods in everyday context and problems*’ (1500) and acknowledges that to make probability meaningful is a challenging task for teaching.

***1.2 Design, manipulatives and probabilities***

Over the last decades there has been a move from traditional mathematical classrooms to student-centred ones where discourse and conceptual development are central (Garfield and Ben-Zvi 2008; Van de Walle, Karp & Bay-Williams, 2019). While exploring probabilities, Sharma (2016), proposes that students should engage with carefully designed sequences of activities that encourage explanation, evidence, creation of examples, generalisation, analysis, predictions, applications, representations of ideas in different ways, and articulation of connections or relationships between probabilities and other topics. In doing so, the design of probabilistic tasks and use of manipulatives are important. Skoumpourdi, Kafoussi and Tatsis (2009) found in their study with 22 preschoolers that probabilistic reasoning is influenced by the nature and structure of the particular task or problem situation. They identified as key factors the meaningful context, the manipulation of concrete materials, rich discussions, reflection and children’s informal knowledge on probability. Falk et al (2012) also address the need for systematic examination of the effect of the structure of the problem when using probabilistic tasks and support through their study with binary choices that young children from the age of 4 can be introduced to probabilities through hands-on playful ways. Furthermore, Schlottmann and Wilkening (2012) underline that task complexity, in relation to linguistic, memory and meta-cognitive demands, can define children’s appreciation of probability. Additionally, Hodnik Čadež & Maja Škrbec (2011) agree that concrete experiences and experimentation are key in teaching probabilistic concepts in preschool children.

Manipulatives play a key role in children’s mathematical understanding as they can be handled in ‘*a sensory manner during which conscious and unconscious mathematical thinking is fostered*’ (Swan & Marsall, 2010, p.14). They have a long history in early childhood mathematics education. Both Montessori (1964) and Froebel (1912) considered manipulatives in their pedagogical approaches, where education was seen as a self-activity including self-discipline, independence and self-direction. For Montessori, children should be encouraged to work independently with manipulative materials that facilitate their learning. Before learning how to write children are first encouraged to outline geometric figures like circle, square, triangle, ellipse, rhombus, and pentagon (Balfanz, 1999). Moreover, the long-term use of the same or similar manipulatives, proposed by Montessori, enables children to abstract the mathematical concepts and make connections (Lillard, 2005). While Montessori focused on sensory education, Froebel (1912) addressed symbolic education (Saracho & Spodek 2006). He designed educational play materials, ‘*Spielgaben*’ (in German play gifts and in English gifts) that embody mathematical ideas such as symmetry, shape, and number, including structural-design toys, pattern recognition and building blocks. According to Manning (2005), the aim of the gifts was to make children feel familiar and comfortable to use, thus, accelerating and enhancing their learning experience. The gifts are underpinned by the cornerstones of unity, respect and play.

Manipulatives are resources that provide opportunities for exploration, discussion, manipulation and conceptualisation. They offer ways of connecting mathematical ideas to real-world experiences (McNeil and Jarvin 2007). They combine senses, action and perception and provide opportunities for scaffolding learners’ modes of representation from enactive, to iconic, to symbolic (Bruner, 1960). In the case of probabilities, these manipulatives could be dice, spinners, cards, board games, tinker cubes, urns, boxes or bags (used as spaces to host a number of items in different ratios or proportions), stories and scenarios, books and visual stimuli, props and tools, toys, that could encourage exploration and discourse around the probable, the most probable, the least probable, the improbable event. For instance, Kinnear and Clark (2014) found in their study with 5 year-olds that picture story books and data modelling tables provided opportunities for the activation of children’s intuitions about chance and probabilistic reasoning.

Nowadays, besides the tangible mathematical manipulatives, there is an increased use and interest in computer-based manipulatives in classrooms. According to Sarama and Clements (2009), these technological manipulatives provide unique affordances for the development of knowledge, they offer great control over direct change, repetition, recording of actions and flexibility in facilitating the development of mathematical ideas. As some of them replicate the concrete manipulatives in a virtual environment, whereas others allow learners to explore concepts that would be difficult to explore in the physical world (Bujak et al, 2013), there is research exploring the benefits of using either or both. For instance, Papadakis, Kalogiannakis, & Zaranis, (2018) found that greek pre-schoolers who used computer-based manipulatives in numeracy outperformed children who had not. More specifically, they found that the type of technological device made a difference in that children who used tablet computers significantly outperformed the children who used personal computers. Likewise, Alcoholado et al (2016), found that when comparing children’s arithmetic solving in Chile through the use of interpersonal computers (one projector with a screen, one computer and one mouse per child), personal computers (netbooks) and pen-and-paper, they showed more increased learning when provided with instant feedback (while using computer manipulatives) as opposed to delayed feedback (while using tangible pen and paper). In UK, Outhwaite, Faulder, Gulliford, & Pitchford (2019) found through their study that combining child-centered, curriculum-based apps with interactive touch-screen tablet technology for children aged 4–5 years old provides an effective means of delivering quality instruction and early math development.

However, the physicality or presence of manipulatives, both tangible or computer-based, is not sufficient in ensuring meaningful learning (Carbonneau, Marley and Selig 2013; Laski et al 2015). Their effectiveness depends on how they are embedded in practice. They do not carry the meaning of the mathematical idea explored; it is the active, sensori-motor engagement of the learners within the wider pedagogical context that does. It is the interplay of the task features, mental representation, and the status of conceptual knowledge of the users that can either hinder or facilitate learning and engagement at different ages (Betsch et al, 2018). Although manipulatives are explored mainly in mathematical areas like numeracy or geometry, they have only recently gained attention in the field of probabilities and statistics in early childhood.

1. **Methodology**

The study took place in 19 public nurseries in Greece from 5 different Provinces, covering both rural and urban areas. These were randomly selected through a draw. Initially, the Greek Ministry of Education, head teachers and parents were informed about the rationale of the study and once they gave their written consent the data collection was initiated based on the nurseries’ routines. Children (*N=*480), aged 4-6 years old, participated in groups of 3, if their assent was confirmed and after ethical implications were acknowledged (EECERA 2015). The SES of the participants was not considered as a variable in this exploratory study but gender was. Participation was counterbalanced in two Conditions; Condition 1 consisted of engagement with tangible manipulatives and Condition 2 with computer-based manipulatives (Table 1). Each Condition was a replication of the other and the rationale of the animal-card guessing task was the same. Children were asked to guess and record what animal was more probable to appear if different sets of cards were turned over and shuffled.

Table 1. Methodological design of the probabilistic task

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| **Probabilistic Task** |
| Condition 1:tangible manipulatives | Task 1 (3:1) | Task 2 (5:1) | Task 3 (4:1:1) |
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| --- | --- |
| j0428345[1] | j0424138[1] |
| j0428345[1] | j0428345[1] |

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|  |  |
| --- | --- |
| j0424138[1] | j0424138[1] |
| j0424138[1] | j0424138[1] |
| j0424134[1] | j0424138[1] |

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| --- | --- |
| j0428345[1] | j0424138[1] |
| j0428345[1] | j0428345[1] |
|  j0424134[1] | j0428345[1] |

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| Condition 2:computer-based manipulatives |

­­­In both conditions, there were 3 tasks with variations in the sample space. Sample space has been identified as one of the key probabilistic constructs (Jones et al 1999) and is the technical term for all the outcomes that are possible in a particular context (Nunes et al 2014)*.* Defining the sample space Ω is crucial to probabilistic thinking because the sample space characterises what is possible and what is not possible besides the likelihood of each event to appear; P (A). In task 1 the sample space was 4 and distributed as 3:1, in task 2 it was 6 and distributed as 5:1 and in task 3 it was 6 distributed as 4:1:1. Thus, tasks 1 and 2 had 2 items and task 3 had 3 items.

Trials were repeated three continuous times in order to see whether there is a learning pattern or whether the outcome of each prediction influences children’s later choices. This principle was used by Falk et al (2012) who also replicated the same trial twice in order to test whether the outcome of each trial influences children’s responses in forthcoming trials. Children recorded on specially designed sheets their predictions and the actual outcome of each trial.

In a separate space or room, children participated in groups of 3. After building rapport with the researcher they were presented with animal cards and were encouraged to count them, turn them around and mix them up. Then, the researcher would ask each child separately: ‘Choose one card’. As soon as the child would select one card, the researcher would ask: ‘If you turn over this card what animal do you think is *most probable* to appear?’. As soon as participants would give an answer and graphically code their prediction on their sheet, they would turn over the card and find out the outcome. While recording the actual outcome the researcher would repeat the same procedure with the second child and in turn with the third child. In Condition 2, through PowerPoint children would see the sets of cards being presented to them, being turned around and being mixed up. This process would be pre-programmed and after children would pick a card, they would use a mouse to click on the chosen card and see the outcome. As in Condition 1 they would record their predictions and outcomes on the pre-designed sheets.

During the task the researcher would keep a neutral stance in the feedback provided by avoiding to give emphasis on the draw outcome. Children were assigned to the tasks in a counter-balanced order and were encouraged to interact with each other. As soon as participation was completed children would­­ return to their classroom activities.

1. **Results**

In making predictions under uncertainty there is no ‘correct’ or ‘incorrect’ answer as there is a high element of randomness. Thus, for the current study, ‘correct’ is coded as the ‘most likely’ animal card and ‘incorrect’ as any other choice. For example, in task 1 the duck would be the ‘correct’ response and the mouse the ‘incorrect’. Children (M = 4.9 years) gave overall 72.8% accurate predictions in task 1, 70.8% in task 2 and 41.7% in task 3. There were 53.5% boys and 46.5% girls and no significant difference in the correctness of predictions was found based on gender, p >.001.

***3.1 The role of tangible vs computer-based manipulatives***

The different types of manipulatives made a difference on children’s predictions. Children made more correct predictions when using tangible (M = .54; SE = .01) compared to computer-based animal cards (M = .46; SE = .01). This difference was statistically significant *t*(4180)= 4.83, p < .001. Children gave more accurate predictions in the case where they interacted and sensed the cards rather than when they used a mouse and a computer screen (Fig. 1).

Figure 1. Number of responses based on the type of manipulative

Like in the case of other mathematical notions, manipulatives as transitional objects that enable thinking and reasoning through action, motion, sensation and perception (Swan and Marsall 2010), enabled children to make inferences and predictions based on precise information. It was the tangible rather than the computer manipulatives that encouraged children to provide significantly higher levels of correct predictions. These findings address the importance of concrete experiences (Hodnik Čadež & Maja Škrbec 2011) and enactive representations (Bruner, 1966) when exploring probabilistic concepts in early childhood. They support previous findings that highlight how the manipulation of concrete materials in a playful context can foster preschoolers’ probabilistic thinking (Skoumpourdi et al, 2009; Falk et al, 2012). However, this preference does not align with findings that favour computer-based manipulatives in other mathematical areas (i.e. Papadakis et al, 2018; Outhwaite et al, 2019).

***3.2 The impact of sample space***

The distribution of the sample space influenced children’s predictions. On average children performed statistically better for tasks 1 (3:1) and 2 (5:1) compared to task 3 (4:1:1). The difference was significant, *t*(4180)= 13.7, p < .01. The sums of two categories of items, in tasks 1 and 2, were easier for children to handle compared to task 3 where there were 3 different categories of animals. The simpler the sample space was the more correct predictions pre-schoolers made (Fig. 2).

Figure 2. Frequencies of responses based on sample space variations

The distribution of the sample space reflects how the complexity of the task might influence children’s probabilistic reasoning. In the case of the more complex distribution it could be that the task features did not match children’s mental representations and status of conceptual knowledge (Betsch et al, 2018). On contrary, the less information was easier for them to engage with. Before the age of about 5 years, children err in complex tasks and answer randomly when they reason about large sets of elements (Girotto et al, 2016). Thus, less data and cues to manage encourage more explicit causal links and reasoning. Furthermore, more information might deviate children’s attention from the underpinning notion of the task or might cause high memory and meta-cognitive demands (Schlottmann and Wilkening, 2012) rendering the activity difficult and hard to participate in.

***3.3 The role of repetition***

The repeated measures didn’t seem to have an impact on children’s predictions. It can be assumed that no learning effect was present with 3 repetitions, *t*(4180) = -0.66, p<0.01. Despite the fact that children would record their predictions and the outcomes of each trial, they did not seem to improve their predictions over time. Thus, the three times repetition did not seem to enable them to make connections and gain deeper conceptual understanding. This replicates the findings that Falk et al (2012) reached in that repetition and the actual outcome did not influence children’s subsequent predictions. Maybe children would need more that 3 repetitions to acknowledge how the most probable outcome can be predicted and estimated each time.

1. **Discussion and conclusions**

Children from the age of 4 were found capable of making inferences and guesses within a probabilistic context. In a repeated-measures task with animal cards children were able to identify the most probable event within given and varied sample spaces, through the use of tangible and computer-based manipulatives. These findings support that children might not have developed advanced computational skills yet (Betsch et al, 2018; Boyer 2007; Denison et al 2013; Girotto et al 2016; Nikiforidou and Pange 2010; Falk et al 2012) but show an intuitive understanding of the likelihood of events. Over the last years such results tend to be consistent in reflecting the separation between probability and the intuition of probability, proposed by Fischbein (1975). These challenge the original claims that children develop probabilistic thinking only after the age of 7 (Piaget and Inhelder 1975). Instead even from early in life infants have been found to form expectations of an event, based on probability and causal learning (Gopnik et al 2013; Denison and Xu 2014; Bulf et al 2011).

A key component in children’s understanding of probabilities relates to the context and the task design (i.e. Skoumpourdi, Kafoussi, Tatsis 2009; Schlottmann and Wilkening 2012; Kinnear and Clark 2014). In the present study, the three main aspects explored were the use of tangible vs computer manipulatives, the distribution of the sample space and the repetition of the task. Each of these aspects have methodological and pedagogical implications for the design, instruction and implementation of basic probabilistic notions in early childhood education. Already in some countries, worldwide, probability literacy, as a knowledge-based and dispositional way of engaging with probability-related information, ideas and methods (Gal 2012; Borovcnik 2016), is introduced within preschool mathematics curricula. As Batanero et al (2016) underline the need for probability literacy has been recognized by educational authorities globally and probabilities are included in curricula and in teacher training. Nonetheless, by having a topic in the curriculum does not automatically ensure appropriateness and effectiveness in teaching and learning. Design and context components are necessary to be taken into account when examining how probabilities can be part of young children’s early formal educational experiences.

In exploring the role of manipulatives in early childhood mathematics, research tends to compare tangible and computer-based ones, aiming to identify which are more effective. Recent studies, predominately in numeracy and arithmetic, have reached the conclusion that computer-based manipulatives and experiences enhance deeper levels of mathematical thinking and learning in young children compared to more traditional means, like pen and pencil (Papadakis et al, 2018; Alcoholado et al, 2016; Outhwaite et al, 2019). However, the current study proposes the reverse but in relation to a different mathematical field; that of probabilities. So far, there has not been any research comparing the use of different manipulatives in young children’s probabilistic reasoning and this study aims to contribute in this gap. It could possibly be the nature of probabilities that might require more sensori-motor experiencing before experimentation within a computer-based environment. Furthermore, despite their increased use, computer-based manipulatives, are sometimes viewed as an extension of the tangible manipulatives (Bujak et al 2013) and thus, their affordances might be restricted. Perhaps in the computer condition children did not experience the meaningfulness of the task and could not make links between their hands-on actions and the notion of chance and likelihood. The fact that the computer presented a 2D dimensional representation of the animal cards could have distracted them from the purpose of the game.

In addition, this study did not explore probabilistic task design as part of instructional or planned activities. There were no learning goals or follow-up activities as part of the research in order to record how children sustain, extend and develop their initial probabilistic responses. This limitation reinforces the argument that the presence and only of the manipulatives is not enough in grasping the underpinned mathematical or probabilistic idea. Instead, it is a matter of how they are incorporated in meaningful activities and contexts that reveal their usefulness and effectiveness (Sarama and Clements 2009; McNeil and Jarvin 2007; Carbonneau et al 2013; Laski et al 2015). Future research could explore how probabilistic manipulatives could be implemented as part of curriculum development and the broader educational context considering the role of the teacher, the role of probabilistic discourse and the role of the children.

The sample space, Ω, is a key construct in understanding randomness (Nunes et al 2014) and a methodological aspect in designing probabilistic activities for children (Jones et al 1999). Every estimation of probability is based on the sample space of the problem. Knowing and understanding the sample space enables someone to estimate the odds, the possibilities and the probabilities. It was found that, when information was less and simpler, children would process and make sense of the probabilistic scenario in more accurate ways. This could be linked to the ‘less is more’ paradigm in that children’s resource constraints limit their ability to use complex hypotheses (Yurovsky et al 2013). Instead, the less information they handle the more they are in the position to maximise the possible combinations.

Repetition was not found to have an impact on the correctness of children’s guesses. Despite the fact that children would record their predictions and actual outcomes, the draw outcome did not seem to influence their predictions in follow-up trials. It could be argued that 3 times for 5 year-olds might not be adequate in establishing a learning pattern. Falk et al (2012) also found an age trend in that children progressively develop to be less swayed by the outcome and to adhere to correct choices. Schlottmann and Wilkening (2012) agree that further investigations are needed to clarify how children use information and update their probabilistic beliefs during repetition. They summarise that children’s judgments show non-normative recency and children learn about probabilities from various forms of frequency data. Thus, the traditional proposition from Montessori and Froebel of using manipulatives repeatedly either as part of sensory or symbolic education in order to develop and master mathematical concepts could be an area for further exploration.

Children reach formal education with prior knowledge and dispositions as well as a range of experiences in probabilistic contexts (Nikiforidou et al 2013). Research is continuous in exploring how to bridge these initial experiences and intuitions of probability (Fischbein 1975) with more computational constructs of probability and randomness in the preschool classroom. In this process of linkage and skill-building, instruction and play, pedagogical approaches, meaningful learning and task characteristics are core. Therefore, the design and context of probabilistic tasks are fundamental in triggering children’s intuitive or analytical or the combination of both ways of thinking. As Jones et al (1999) have stated, instructional activities can shape children’s understandings or misunderstandings of basic probabilistic notions. If these are not approached according to the child’s ‘forms of thought’ (Bruner, 1960) during early childhood then there might be later challenges as the child grows older. Nevertheless, Watts et al (2018) would argue that this correlation is not straightforward and depends on the learning environment, context and content of the younger and older ages.

To conclude, probabilities are gradually becoming an integral part of early childhood curricula. Children’s probabilistic competence is more profound than previously thought and there is increasing interest in exploring the most effective ways of introducing probabilistic notions from early childhood, aiming at probability literate children. In this direction, this study identified that the use of manipulatives, especially tangible ones, and the simple and less overloaded sample space can provide 4-6 year olds with opportunities to predict the most probable event in random draws.

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