

**DEVELOPING MATHEMATICAL IDENTITY AND ‘UNDERSTANDING  
MATHEMATICS IN DEPTH’: CONCEPTIONS OF SECONDARY  
MATHEMATICS TEACHERS ON AN IN-SERVICE SUBJECT  
KNOWLEDGE ENHANCEMENT COURSE IN ENGLAND**

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*Conceptions of ‘understanding in mathematics in depth’, and its importance for mathematics teachers, are issues of current debate with implications for teachers’ in-service training courses. This paper focuses on a group of nine mathematics teachers who had completed a government funded in-service programme which aimed to strengthen teachers’ understanding of fundamental mathematics. They were non-specialists, teaching mathematics ‘out-of-field’. Qualitative data was drawn from semi-structured interviews with the teachers, and an in-depth case study of one of the sample. Analysis of data showed that for these teachers, growth in knowledge was located mostly within subject matter knowledge (SMK). ‘Understanding mathematics in depth’ (UMID) was articulated as ‘knowing why’ and ‘being able to communicate’. When analyzing themes highlighted by the teachers, two new key themes emerged relating to shifts in identity and reconstruction of subject knowledge. The findings offer important messages for teacher education policy and practice, with respect to the value of substantial in-service training programmes.*

**INTRODUCTION**

“It gives you the confidence to be a bit freer in the classroom to allow the pupils to investigate things themselves, rather than come in and say ‘this is how you do it, it is just because it is’. That’s the big difference in my teaching.”

This is the voice of ‘John’, a non-specialist secondary mathematics teacher. This paper reports on a study of teachers like John, and how their knowledge development gained through in-service provision impacts upon their self-perception and their teaching. The paper gives a brief overview of the study and its context; how the study relates to existing related research; outcomes of the study with a focus on the voices of the teachers; and implications for policy and practice in mathematics teacher in-service provision.

**RESEARCH CONTEXT**

The Mathematics Development Programme for Teachers (MDPT) is a part-time course for qualified secondary teachers who are not mathematics specialists. It arose following the recommendations of the Smith Report (2004) that significant opportunities for professional development should be made available to serving mathematics teachers, in particular those who are non-subject specialists. The course is fully funded by government. At the time this research was carried out, the course duration was a maximum of 30 taught days and 10 school-based development days, taking place over four terms. Both the specification and the title of the course have evolved; it is currently known as the Teacher Subject Specialism Training (TSST) course (National College for Teaching and Leadership, 2015). Content of the course includes exploration of key aspects of the school mathematics curriculum, and the development of teachers’ pedagogical approaches. Three strands

interweave throughout the course: development of teachers' subject matter knowledge (SMK), of their pedagogical content knowledge (PCK), and of their understanding of the links between research and practice in mathematics education. The existence of this course gives formal recognition to a problem in England which has been known for a long time, namely that much school mathematics at secondary level is carried out by non-specialists. Smith (2004) identified a shortage of around 3,400 mathematics teachers. More recent government figures (Department for Education, 2012) indicate that around 27% of those teaching secondary mathematics in England do not have a post A-level qualification in the subject. 'Upskilling' non-specialist mathematics teachers, many of whom have significant teaching experience, enhances the expertise of the existing workforce in teaching. It is more cost-effective than training new mathematics teachers. It also breaks the vicious circle of a poor supply of teachers at a different point (Smith, 2004). The way in which one might define a specialist mathematics teacher is open to discussion, and The Royal Society (2007) has called for consensus on such a definition, to facilitate clearer information gathering and workforce planning.

### **Features of the course**

On the MDPT course discussed in this paper, key aspects of the school mathematics curriculum are explored, with explicit links being made between the strengthening of participants' personal subject knowledge, and the development of their knowledge and understanding of effective pedagogical approaches. Common misconceptions in mathematics topics are also part of the content taught. Not only should this enhance teachers' understanding of their own students' learning and the barriers to it, but it is also intended to provide a 'safe' (i.e. personally unthreatening) context for their own mathematical misconceptions and partial understandings to emerge. Thus they should be able to move on and develop their own knowledge and understanding. Course content and teaching is underpinned by, and related to, relevant research. Course participants may opt to undertake formal academic assessment and achieve a Postgraduate Certificate in Mathematics Education. This is strongly recommended due to the depth of learning that it affords, and the majority of participants do in fact complete this.

Another important strand of the MDPT is to encourage participants to identify with the wider mathematics community, and opportunities are provided for them to take part in local subject association activities such as children's masterclasses and subject meetings. All course participants register with the NCETM (National Centre for Excellence in Teaching Mathematics), and all use the NCETM online self-evaluation tools to help them to establish their subject knowledge training needs and to track their progress in development of subject and pedagogical knowledge.

### **KNOWLEDGE FOR MATHEMATICS TEACHING – LOCATION IN THE FIELD**

Key contributors to research into mathematics knowledge for and in teaching, including Shulman (1986), Ball et al. (2008), Baumert et al. (2010) are discussed elsewhere (Stevenson, under review). The MDPT is a relatively new addition to the UK mathematics education landscape, and as yet there has been little research on this programme. Work in this study makes a contribution to addressing that gap. Crisan and Rodd (2011) discuss the effects of the MDPT course in developing participants' evolving identities as mathematics teachers within a community of practice. Vale, McAndrew and Krishnan (2011) report on a course in Australia, designed for 'out-of-field' (i.e.

non-specialist) mathematics teachers. They note that a significant length of programme is important for teachers' mathematical development, as is the quality of relationships established with colleagues whilst they are on the course. They also note the impact of 'positioning practicing teachers of secondary mathematics as learners' (p. 209). Graven (2004) studies non-specialist mathematics teachers in South Africa on an in-service training course. She highlights the importance in the teachers' discourse of confidence growth, and also the importance attached to the idea of playing a fuller part in a mathematics education community. Graven notes that confidence develops over time, and asserts that "mastery involves the insight to know when you do not know, the confidence to admit to this, and the ability to access the necessary information..." (p. 207). Both Graven (2004) and Crisan and Rodd (2011) discuss teachers' learning on professional development programmes as induction into a community of practice (Wenger, 1998).

Restructuring of knowledge is a key component of knowledge development for teachers in service. Ruthven (2011) argues that in learning to teach, the restructuring - or decompressing - of existing mathematical knowledge may have a more important part to play than the acquisition of wholly new concepts. This can often also include a challenge to teachers' beliefs and thus some reconstruction of identity (Ruthven, op.cit. and Hodgen, 2011).

In the study reported here, it was important to keep in mind both the distinction between, and the interplay of, subject matter knowledge (SMK) and pedagogical content knowledge (PCK) (Ball, 2008). Ma (1999) calls for integration and simultaneous development of subject knowledge and pedagogical knowledge in teacher preparation courses, as do Davis and Simmt (2006). Ma's construct of 'profound understanding of fundamental mathematics' (PUFM) provides an important way of conceptualising teacher knowledge. I understand PUFM to be strongly related to SMK. However, PUFM is developed through the act of teaching, and so it also links to PCK and to Ball's specialist content knowledge (SCK).

## **DATA COLLECTION AND ANALYSIS**

A sample of nine current and former MDPT teachers from one university were interviewed, with participants drawn from each of the 2007-08, 2008-09 and 2009-10 cohorts. The semi-structured interviews took around 30 minutes each, and were digitally audio-recorded and later transcribed. Questions probed, *inter alia*: participants' experience of mathematics during their training/course and how this prepared them for their own teaching; topics which the respondent understands well, and where this understanding came from; what 'understanding maths in depth' means to the participant.

Four stages of analysis of transcripts were undertaken. At the first stage, initial identification of comments seen as significant was made in an open, unstructured way. This was possible – and justified - due to the researcher's extensive experience in mathematics education. Thus the teachers' stories began to emerge. Following this at the second stage, emerging codes were identified from the data. The third stage involved tabulation of participants' responses to individual questions, and a cross-participant question-by-question analysis. At the final stage, transcripts were re-read holistically in the light of prior coding and analysis, and attention was shifted back to the story being told by each teacher.

What follows are some examples of teacher responses in the interviews. There is a focus upon hearing the voices of the teachers and of John, the individual in the case study.

### **What is UMID and how is it developed**

The majority of respondents conceived ‘understanding mathematics in depth’ as ‘knowing why’ and also in terms of being able to communicate ideas to others. For example:

“I feel that actually shows true understanding if you can explain it to others. Because that’s reinforcing it for yourself as well as being able to show them that you know your ideas”. (Int 2, MDPT)

This is supported by work by Adler et al. (2013) whose study of preservice teachers reveals that their perceptions of ‘understanding mathematics in depth’ strongly feature communication and mathematical reasoning or knowing why.

Various comments were made relating to immersion in the subject, and spending time working on mathematics. This supports Watson and Barton’s (2011) ideas of teachers enacting mathematics, and being involved in the process of mathematics. For example,

“I get deeper and deeper knowledge of it anyway because you read the books and you talk to people and you live in it. Because maths is a foreign language isn’t it? And the best way to learn a foreign language is to go and live in the country and that’s what I do”. (Int 18, MDPT)

Results of the study show that MDPT teachers focus upon a growing confidence in teaching mathematical material that they previously were not confident with. Thus we see a growth in mathematical content knowledge and specialised content knowledge rather than a growth in general teaching strategies. They also comment upon alterations to their perceptions of their professional selves, seeing themselves in a different way – as maths teachers – as a result of following the course. This can be understood in terms of identity, and is echoed in the findings of Crisan and Rodd (2011) in their study of MDPT teachers. Graven (2004) also comments upon the shift in teachers’ perceived identities, from ‘teacher of mathematics’ to ‘mathematics teacher’.

### **Emerging themes**

At the final stage of data analysis, two interesting themes emerged, in areas which had not been probed explicitly in the interviews:

#### **1) Growth in knowledge and confidence can alter one’s perceived identity**

Several respondents commented in this area, for example,

“I am a much more confident teacher from being here and doing all the different things we have done, taking them back with me and I actually feel like part of the mathematics department...and I am comfortable with it, whereas two years ago if someone had said you will be teaching statistics in year 11, I would have looked at them and laughed..” (Int 5, MDPT)

This teacher also commented on the benefits of collaboration with peers in deepening her own understanding. She clearly felt a strong sense of being as a maths teacher now. A similar effect can be seen with another interviewee, who worked in special education and who commented upon the effects upon him of following the MDPT:

“In the classroom [the course has] helped tremendously. And what it has done as well, is when I go to Heads of Departments meetings... I’m kind of treated as an equal even though I’m not in the true sense of

the word, a mathematician...I think if I had not done that course...I don't think I'd have had the same respect."

"...what it's [the course] done is it has given me the confidence to try new things."

"...I now read stuff in the bath about mathematics and stuff like that which I wouldn't have done." (Int 16, MDPT)

## **(2) Reconstructing existing knowledge can result in transformation of teaching approach**

An interviewee reports a change of teaching approach as a result of her experience on the MDPT – she notes that the course ‘stripped back’ basic ideas of maths (c.f. Adler and Davis (2006) ‘decompression’), so that now she understands learners’ misconceptions better. She reports that she was already confident in the mathematics herself, but now she opens up more contexts and applies it differently, and is more enthusiastic. Another interviewee comments on his experience of reconstructing mathematical knowledge:

...and I would read through it and I would say, ah, I didn't really realise that you could do that. Even as an adult because I would be in my 30's then and it was this kind of hidden thing that I hadn't discovered until I was grown up. (Int 12, MDPT)

Similarly, another interviewee states that MDPT enabled her to see links between different aspects of mathematics. She was teaching various techniques but did not have a full understanding; now she has a fuller picture and sees the connections between concepts. This theme emerges strongly from the responses of MDPT teachers. Lacking sufficient preparation in subject knowledge and pedagogy, many had originally adopted a procedural approach to teaching mathematics. Engagement with the MDPT course provided opportunities for challenges to their understanding of key concepts, and a subsequent re-structuring and deepening of their knowledge. This then enabled them to develop their teaching approaches significantly.

### **Case study**

I now consider in more detail some responses of one of the teachers, who I shall refer to as John, to show more clearly how the above emergent themes are located in his discourse. At the time of the interview, John was in his late twenties and had been teaching for four or five years. He followed the MDPT course in 2009-10. He had originally trained and worked as a Business Studies teacher, but at the time of the interviews was teaching mathematics at an urban comprehensive school in an economically deprived area of Merseyside.

John clearly articulates the change in his teaching approach that took place as a result of his experience on the MDPT. He describes how when he began teaching mathematics, he was nervous about being asked questions, and used Ten Ticks (2011), a resource which provides multiple pupil worksheets containing hundreds of short procedural questions and exercises (my italics):

"There are often things that I had never come across. You know, when... because my training was business studies and *to suddenly be thrown into a classroom and say right today you are now a maths teacher* without understanding the courses, anything like that and initially *for your first 3-6 months it is just making sure that every lesson you go to you have got something for the kids to sit down and do in a way that they don't ask you many questions*, Ten Ticks to me in the first three months was the greatest invention of all time because I could go in and hand them out and I could have a pile of Ten Ticks ready for them. *I didn't want to be asked any questions, you know...*"

He describes how formerly because of his own limited subject knowledge he lacked confidence and adopted a prescriptive approach to teaching, but that now he is able to use more open, investigative approaches to teaching and learning. As a result of growth of his own confidence in the subject, he has the confidence to allow pupils to explore, and does not feel he has to control everything:

“And it is about them investigating it and slowly but surely being able to pull ideas out of kids and I think what that session that we did last time gave me a clear understanding of why things work and *it is actually things I often didn't understand* and didn't have a clear understanding of I now know how to think about those things now and I know how to draw it out of pupils rather than going in and saying you know, this is how it works because it does.“

John moves on to talk about the importance of passion for the subject, and how he is trying to nurture this in his teaching.

“...understanding maths in depth develops in living with or spending time thinking about mathematical ideas, I think again because once you start enjoying something then you want to do more of it. Someone like [*names one of the course tutors*] is a great example, you know, everything he does is a mathematical... it becomes part of your life doesn't it.

...which I think for your subject that is such an important thing, you know, I think if you look at outstanding teachers from OFSTED you know one of the underlying things is a passion for your subject...and the only way you're going to get passion for your subject is if that subject becomes a part of you“.

In the case of John, we see clearly that his teaching approach has been transformed as a result of his engagement with the MDPT and the reconstruction and development of his own knowledge that this has entailed. Theme 1 (*Reconstructing existing knowledge can result in transformation of teaching approach*) clearly emerges from his discourse as he talks with enthusiasm about his role and his work. John says that although he does not think of himself as ‘an amazingly better mathematician’, he has a lot more confidence teaching the subject and allowing pupils to investigate. This and his other remarks suggest growth in his confidence and knowledge, implying he is a better mathematician at the school level and that his perception of himself as a mathematics teacher has developed. Thus theme 2 (*Growth in knowledge and confidence can alter one's perceived identity*) is implied by his discourse. In the case of John it is possible to see the hallmarks of experiences which are shared by his colleagues in the MDPT sample.

## DISCUSSION

Themes 1 and 2 relate strongly to changes or transformation that can take place when an established teacher encounters new concepts, or familiar concepts presented in a new way, and is offered the opportunity to try out new ways of thinking and working. They highlight the potentially transforming effect of well-designed and substantial in-service training programmes.

All of those on the MDPT are serving teachers, some with many years' experience of teaching but in subjects other than mathematics. They have moved into teaching mathematics relatively recently. Many admit that before the MDPT course the only strategies for teaching mathematics that they knew about were those that they had previously encountered as school students themselves, which were often very procedural in approach. The case study of John is a good illustration of transformation of teaching approach brought about by a reconstruction and development of his own knowledge.

Several participants commented on changes to their knowledge and confidence, linking this to an altered self-image. Several of the MDPT respondents comment that they have changed their teaching approaches as a result of their improved or altered understanding of the mathematics involved. In particular, they comment upon making more links between concepts, using more varied applications and contexts, and in particular about giving their students opportunities to explore, investigate and think mathematically, rather than attempting to direct and control their thought. Several make comments about not having thought about it this way before, did not previously see the links, didn't realise you could do it that way, etc. There is clear evidence of a reconstruction of knowledge happening here. These teachers are not starting from the beginning – they have a knowledge base, but it is incomplete and may involve misconceptions.

Watson's (2008) assertion that non-specialist teachers need "more personal experience of the mathematical canon" (p. 5) is relevant here, as are Davis and Simmt's (2006) comments about teacher education courses needing to involve mathematics that is "new to the do-ers" (p. 316). It is evident from the interviews in this study that having been given this experience on the MDPT, non-specialist teachers can then go on to change their teaching approach quite significantly.

It is important to note some limitations of this research. Firstly, the sample was from one university only, and whilst providing a helpful illumination of practice, this limits the extent to which wider generalisations can be made. Wider research would be beneficial. Secondly, it is significant that at no stage during this study was any attempt made to actually measure objectively teachers' subject knowledge. All the data and results are obtained from teachers' *own perceptions* of their knowledge, understanding and confidence. It would be very interesting to capture growth in knowledge and then to compare this with teacher perceptions. Warburton (2014) suggests an approach to measuring teacher knowledge using the mathematical knowledge for teaching (MKT) items developed by the University of Michigan (Ball, Hill and Bass, 2005).

## Conclusion

Much of teachers' CPD provision in England is in the form of short, one-day courses, which although helpful are necessarily limited in scope. My findings in this paper show that engagement over a period of time in an in-depth course, can produce transformative results. The message to policy-makers is that this can be achieved at relatively low cost, meeting a national need to provide more trained mathematics teachers. It is possible for non-specialist teachers to develop into confident mathematics teachers.

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