A Dynamic Game of Mobile Agent Placement in a MANET

Ekaterina Gromova, Dmitry Gromov, Nikolay Timonin
Faculty of Applied Mathematics and Control Processes
St. Petersburg State University
St. Petersburg, Russia
Email: e.v.gromova@spbu.ru, dv.gromov@gmail.com

Anna Kirpichnikova, Stewart Blakeway
Department of Mathematics and Computer Science
Liverpool Hope University
Liverpool, UK
Email: kirpica@hope.ac.uk, blakews@hope.ac.uk

***Abstract*—In this paper, we describe a novel game-theoretic formulation of the optimal mobile agents placement problem which arises in the context of Mobile Ad-hoc Networks (MANETs). In particular, we consider two classes of multistage games: sequential and simultaneous. For such games, the definitions of the Nash equilibria and the cooperative solution are given. The described games exhibit a number of interesting features. For instance, the Nash equilibrium may turn out to be unattainable in both a simultaneous and a sequential game. In this case, the game dynamics may exhibit the behaviour similar to that of a limit cycle albeit in a discrete space. A modelling environment for the analysis of different strategies of the players was developed in MATLAB. The programme generates various game situations and determines each players move by solving respective optimisation problems. Using the developed environment, two specific game scenarios were considered in detail.**

***Keywords*–MANET; Dynamic games; Multistage games; Drone placement; Graphs; Nash equilibria**

I. Introduction

Recently, there has been a growing interest in the qualitative analysis and performance optimisation of Mobile ad-hoc networks (MANETs). A MANET is a network with no predefined infrastructure that consists of a collection of wireless mobile nodes/agents [1]. MANETs play an increasingly important role and have many applications such as disaster recovery after a natural catastrophe, for instance after an earth quake, search-and-rescue operations and so forth [2]. One of the biggest problems associated with the operation of a MANET is the need to share/use limited resources for transmission of radio waves. Inappropriate use of the resources can cause a severe degradation in network performance [3].

When the number of nodes wanting to transmit data increases some of the routing paths become congested and performance drops [4], [5]. In some cases the obstacles on the ground (lakes, buildings, etc.) cause link breakages. In order to mitigate against the congestion of a given link, or to fix link breakage, drones can be used.

Introducing mobile agents (drones) and finding the best possible locations for them is the focus of this research. Typically, the number of drones is rather limited so they should be placed in order to maximally increase the performance of the network. Apart from that, a common situation is that there are several groups of nodes which are deployed to solve their specific tasks. These nodes may or may not be able to communicate with each other. The latter case corresponds to the situation when, e.g., the nodes from different groups use different frequency ranges [6].

We address the described problem in a decentralised manner. That is, we assume that each group of nodes has a single control centre which is in possession of a single drone. The goal of the control centre is to place the drone in order to maximise the performance of the subnetwork formed by own nodes.

It turns out that Game theory lends itself perfectly to addressing the described problem. Game theory is a powerful tool to study situations of sharing limited resources, and it is dealing with finding the best actions for individual decision makers (players) and finding the best available outcome [7], [8]. Using the game-theoretic methods one can explicitly design and analyse strategic choices and model the decision-making process for the player according to their own interests. Much of the research conducted in the application of the game theory to the area of mobile networks is related to malicious/selfish nodes detection [9], [10], [11], [12], [13].

In this paper, we take a different approach and apply game theory to a special class of network optimization problem as described below. Note that there are a number of results on games as applied to networks (see, e.g., [14] and [15]). However, this particular problem statement appears to be novel thus opening wide opportunities for further research.

This paper is structured as follows: in Section II, we give a formal description of the considered objects and introduce the mathematical notation that will be used hereafter. In Section III we present the game-theoretic formulation of the drone placement problem. Section IV contains a number of numerical examples, while in Section V we draw conclusions and outline the directions of future research.

II. Problem statement

A. Two types of communication networks

We consider the set $\mathcal{N} = \{1, \dots, N\}$ of N players, each player $i \in \mathcal{N}$ has a non-empty set $\mathcal{M}_i = \{1, \dots, M_i\}$

of agents. Each agent $a_i^j \in \mathcal{M}_i$, $i \in \mathcal{N}$ is characterised by a pair of coordinates (x_i^j, y_i^j). We assume that the agents can be located at the vertices of a uniform tiling of a connected subset of the Euclidean plane $C \subset \mathbb{R}^2$. In practice, one considers three kinds of uniform tiling: the triangular, the Cartesian (integer), and the hexagonal tiling which are formed by unit equilateral triangles, squares, and hexagons. Henceforth we will denote the set of all admissible coordinates by $W \subset C$. In the case of Cartesian grid, the coordinates of admissible points are the pairs of integer, i.e., $W \subset \mathbb{N}^2 \cap C$.

The agents form a communication network whose structure is determined by the spatial alignment of agents. While the very nature of a MANET suggests a dynamic topology, we look at a snapshot of the network at a given point in time. Thus, we assume that the communication network formed by the respective agents does not change with time. In the following, we will consider two types of communication structures: separate and joint use of communication infrastructure. We denote these two types of communication networks by $\mathbf{S}$ (separate) and $\mathbf{J}$ (joint). Below we consider these two cases in more detail.

$\mathbf{S}$-*network:* The communication network consists of N disjoint graphs $\mathcal{G}_i^S = (\mathcal{V}_i^S, \mathcal{E}_i^S)$, $i \in \mathcal{N}$, where the sets of vertices $\mathcal{V}_i^S = \{v_i^k = (x_i^k, y_i^k)\}_{k=1}^{M_i}$ are the collections of the Cartesian coordinates of agents of player i; the sets of edges $\mathcal{E}_i^S = \{e_i^{k,s} = (v_i^k, v_i^s) \in \mathcal{V}_i^S \times \mathcal{V}_i^S : \mathrm{dist}(v_i^k, v_i^s) = 1\}$ are the sets of all pairs of agents for player i that are at a unit Euclidean distance from each other. Note that the coordinate grid is such that the distance between two neighbouring points is always equal to 1. We assume that each graph $\mathcal{G}_i^S$ is connected, i.e., there is a connected path between any two agents of a player. Note that there are no connections (links) between the elements of different subgraphs, i.e., the agents of one player do not participate in the transmission between the agents of another player.

$\mathbf{J}$-*network:* This case corresponds to a single communication network whose communication structure is modelled by the graph $\mathcal{G}^J = (\mathcal{V}^J, \mathcal{E}^J)$, where the set of vertices $\mathcal{V}^J = \{v_i^k = (x_i^k, y_i^k)\}$, $i \in \mathcal{N}$, $k \in \mathcal{M}_i$ is the collection of the Cartesian coordinates of all agents; the set of edges $\mathcal{E}^J = \{e^{k,s} = (v_i^k, v_j^s) \in \mathcal{V}^J \times \mathcal{V}^J : \mathrm{dist}(v_i^k, v_j^s) = 1\}$ is the set of all pairs of agents that are at a unit Euclidean distance from each other. As in the previous case, we assume that the graph $\mathcal{G}^J$ is connected, i.e., there is a connected path between any two agents.

Note that in both cases the coordinates of agents of two different players can coincide. While in the case of the $\mathbf{S}$-network this would mean that the graphs may overlap (they can be considered to lie in different layers), for the $\mathbf{J}$-network this would imply that there is no direct connection between two agents located in the same point. This can be relaxed by defining the set of edges to be the set of all pairs of agents at the distance less or equal to 1, i.e., $\tilde{\mathcal{E}}^J = \{e^{k,s} = (v_i^k, v_j^s) \in \mathcal{V}^J \times \mathcal{V}^J : \mathrm{dist}(v_i^k, v_j^s) \le 1\}$.

Note also that the choice of the grid can influence the connectivity of the respective graphs and the related characteristics. While any agent located on a hexagonal grid can have at most adjoin 3 edges, for the triangular grid this number can be up to 6.

B. Graph theoretic ingredients

Before proceeding to the game-theoretic formulation of the problem we present a couple of facts from graph theory which will be used hereafter.

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a graph and s be an agent. We define the union $\tilde{\mathcal{G}} = \mathcal{G} \cup s$ to be a new graph with an extended set of vertices $\tilde{\mathcal{V}} = \mathcal{V} \cup s$ and the accordingly adjusted set of edges $\tilde{\mathcal{E}}$. The union operation can be obviously extended to the case $\mathcal{G} \cup S$, where S is a set of agents $S = \{s_i\}_{i=1}^k$. Note that $\tilde{\mathcal{G}}$ can be disconnected.

The diameter of the graph $\mathcal{G}$, denoted by $D(\mathcal{G})$, is the maximum among all shortest paths between the agents in the graph, $D(\mathcal{G}) = \max_{(v_i,v_j) \in \mathcal{V} \times \mathcal{V}} d(v_i, v_j)$, where $d(v_i, v_j)$ is the graph distance between two vertices v_i and v_j which is defined as the minimum length of the paths connecting them. If no such path exists, $d(v_i, v_j) = \infty$.

Since each graph $\mathcal{G}_i^S$ is connected, for the integer grid the diameter can be roughly estimated as

$$2(\lfloor \sqrt{M_i} \rfloor - 1) \le D(\mathcal{G}_i^S) \le M_i - 1.$$

Obviously, while the upper bound does not depend on the type of the grid, the lower bound does. Namely, it is minimal for the triangular grid and maximal for the hexagonal one.

Furthermore, let $\mathcal{V}_i$ be a disjoint partition of the set of vertices $\mathcal{V}$ of the graph $\mathcal{G}$. We define the diameter of the graph $\mathcal{G}$ with respect to $\mathcal{V}_i$ as $D(\mathcal{G}, \mathcal{V}_i) = \max_{(v_i,v_j) \in \mathcal{V}_i \times \mathcal{V}_i} d(v_i, v_j)$. That is to say, when computing $D(\mathcal{G}, \mathcal{V}_i)$ we consider only the paths between the pairs of vertices from $\mathcal{V}_i$. However, the intermediate vertices in the respective paths are not required to be in $\mathcal{V}_i$.

C. Mobile agent: drone

Each player has a mobile agent (drone) q_i at their disposal. The drone is placed at any admissible point in W. While all the agents have fixed coordinates, the position of the moving agent q_i, $i \in \mathcal{N}$ can be changed during the game. The link between the moving agent and another agent is defined similarly to the links between the stationary agents. The moving agent q_i can establish a link with any other moving agent q_j and with any stationary agent $v_i^p, p \in \mathcal{M}_i, i \in \mathcal{N}$.

At the initial time all drones are assumed to be located at some initial position q^o. We also assume that the Euclidean distance between q^o and the agents is greater than 1, i.e., $\mathrm{dist}(q^o, v_i^k) > 1 \, \forall i \in \mathcal{N}, k \in \mathcal{M}_i$. That is to say, the initial position of the drones is such that they cannot establish communication with the agents of any other player.

Finally, we denote the set of all drones by $Q = \{q_i\}_{i=1}^N$ and the set of all drones except the ith one by $Q_{-i} = Q \setminus \{q_i\}$.

III. Game Formulation

A. Optimisation problem

For any type of communication structure the ith player solves the following optimisation problem:

$$\bar{q}_i = \underset{q_i \in W_i^*}{\arg\min}\, D(\mathcal{G}_i^*, \mathcal{V}_i), \tag{1}$$

where $\mathcal{G}^*$ is the extended graph and $W_i^* = W \setminus Q_{-i}$ is the set of admissible locations for the ith drone. For the case of **S**-network, $\mathcal{G}_i^* = \mathcal{G}_i^S \cup Q$, for the **J**-network: $\mathcal{G}_i^* = \mathcal{G} \cup Q$. This can be interpreted as follows: at each step the ith player aims at placing its moving agent q_i such that it minimises the diameter of the respective graph computed for all player's agents. Note that the drones themselves are not considered when computing the diameter of the graph as they are not assumed to be the sources of any useful information, but merely transmit the information packages between the agents. In this statement, we assume that each drone can be used by the agents of any player. The set of admissible locations W_i^* is introduced to avoid the collision of two drones.

Accordingly, the pay-off (utility) function of the ith player is defined as the difference between the diameter of the respective graph before placing the drone and afterwards:

$$H_i(\mathcal{G}, Q) = D(\mathcal{G}_i^*, \mathcal{V}_i) - D(\bar{\mathcal{G}}_i^*, \mathcal{V}_i) \geq 0, \tag{2}$$

where $\bar{\mathcal{G}}_i^* = \mathcal{G}_i^S \cup Q_{-i} \cup \{\bar{q}_i\}$ or $\bar{\mathcal{G}}_i^* = \mathcal{G}^J \cup Q_{-i} \cup \{\bar{q}_i\}$, respectively. Note that the pay-off function is always non-negative.

We will modify this definition for the multi-staged game in the subsequent section.

B. Game aims and strategic (normal) form

The idea of the game is that each of the players aims to place its moving agent such that it minimises the maximal distance between the player's agents while taking into account the existing communication infrastructure. The degree by which the player minimises the mentioned distance is captured by the pay-off function (2). Thus the goal of the player is to maximise its pay-off function. The game finishes if none of the players can maximise further the respective pay-off function.

The game is now formulated in normal form, [16], i.e., the game is a triple

$$\Gamma = \{\mathcal{N}, S, H\},$$

where $\mathcal{N}$ is the set of players; $S = \prod\limits_{i \in \mathcal{N}} S_i$ is a Cartesian product of strategy sets S_i and S_i is the set of available strategies for player i; H is a vector-valued function such that for any given communication infrastructure $\mathcal{G}$, $H_i : \mathcal{S} \to \mathbb{R}$ is the utility (pay-off) function for player i.

The size of the action set of each player is limited from above: $|S_i| \leq |W| - N + 1$. This estimate shows that our game is discrete and finite. However, the above estimate can be greatly improved. For instance, for the rectangular grid and **S**-network the number of all meaningful actions satisfies

$$4\lfloor\sqrt{M_i} - N + 1\rfloor \leq |S_i| \leq 2(M_i + 1),$$

where when computing the lower bound we took into account the requirement that two drones cannot occupy the same location.

C. Multistage Game

The formulated game is a game with complete information. By the latter we mean that all the players have all the required information about the current and the previous network states, and all the elements of the game are common knowledge. Furthermore, due to the obvious restrictions we confine ourselves to the class of pure strategies.

There are two types of multistage games: simultaneous and sequential ones [17]. In a *simultaneous game*, the players choose their actions simultaneously and without having knowledge of the choices made by other players. In a *sequential game*, the players take their decisions in a certain, a priori fixed order. Thus, the order in which players choose their actions is a cruicial parameter of the game. Intuitively, the player that is the last to decide has an advantage of the others.

Below, we will mostly concentrate on the sequential version of the game. Let $\sigma : \mathcal{N} \to \mathcal{N}$ be a bijective map (permutation of $\mathcal{N}$) defining the sequence of moves. We will refer to σ as the move sequence. At each stage, the players make their choices according to this sequence, i.e., first moves the player $\sigma(1)$, then $\sigma(2)$ and so forth. Each player places its drone in order to minimise the diameter of the respective graph. Initially, all drones are in the starting location $Q^o = \{q^o, \ldots, q^o\}$. When the ith player moves the set Q updates: $Q = Q_{-i} \cup \{\bar{q}_i\}$, where $\bar{q}_i$ is the solution of the respective optimisation problem (1). Note that the decisions taken by subsequent players may change the pay-off function of the player that made its move before. Therefore, the final value of the pay-off function is computed at the end of the stage, when all players have made their moves.

D. Solutions and Equilibria

We are interested in two particular types of solutions to the considered game: a cooperative solution and a Nash equilibrium solution. Below, we give a formal definition of these concepts for the case of **S**-network. All formulated results will hold, *mutatis mutandis*, for the case of a **J**-network.

Definition 1. The solution $Q^{NE} = \{q_i^{NE}\}_{i \in \mathcal{N}}$ is said to be the *Nash equilibrium* solution if for any $i \in \mathcal{N}$ the following holds:

$$H_i(\mathcal{G}, Q^{NE}) \geq H_i(\mathcal{G}, Q_{-i}^{NE})$$

where $Q_{-i}^{NE} = \{q_j^{NE}\}_{j \neq i} \cup \{q_i\}$, $q_i \in W_i^*$.

This condition can be reformulated in terms of graph diameters:

$$D(\mathcal{G}_i^S \cup Q^{NE}, \mathcal{V}_i) \leq D(\mathcal{G}_i^S \cup Q_{-i}^{NE}, \mathcal{V}_i),$$

In plain words, this means that any player cannot improve its payoff function (i.e., decrease the diameter of its graph) by unilaterally changing the position of its drone.

Definition 2. The solution Q^C is said to be the *cooperative solution* if it minimises the sum of all individual payoff functions:

$$Q^C = \arg\min_{Q \in W^*} \sum_{i \in \mathcal{N}} D(\mathcal{G}_i^S \cup Q, \mathcal{V}_i),$$

where $W^* = W_1^* \times W_2^* \times \ldots \times W_N^*$ is the set of all admissible control actions.

Note that the cooperative solution always exists, as follows from the finiteness of the set of actions. Obviously, it can be non-unique. The case of Nash equilibrium is, however, more subtle.

While our dynamic game is represented in a strategic (normal) form, because of the presence of sequential decision-making, an alternative representation highliting the sequence of the moves can be used. The sequence of moves is thus represented in a game-tree form, which is called an extensive form. It is known that every finite extensive-form game with perfect information has a pure-strategy Nash equilibrium [17]. However, the existence of the Nash equilibrium does not imply that the game has a finite extensive-form representation.

That is to say, even the Nash equilibrium solution exists, it may happen that it cannot be reached by any sequence of players' moves. Such situation occurs when the Nash equilibrium solution contains certain configuration of drones which we will call *coherent structures*. One typical example of a coherent structure is shown in Fig. 1. In this figure, 3 drones form a bridge, the stucture which cannot be reached without some extra coordination between players. When the Nash equilibrium cannot be reached, the game will have a periodic solution. This is similar to the situation observed in the theory of dynamic systems when a stable limit cycle surrounds an equilibrium point.

IV. NUMERICAL EXAMPLES

The simulation of the multi-staged game for the **S**-network has been implemented in MATLAB for two players. For testing purposes the world was restricted by an 8×8 rectangular grid. A number of restricted areas were introduced, those which can be thought of as obstacles which prevent player placing agents, i.e. where there might be a lake or building. (see Figures 2, 3). Only mobile agents are allowed to be placed there. For each player, the placement algorithm places a predefined number of nodes randomly on the grid whilst ensuring a connected graph. Then the adjacency matrix $A = [\alpha_{ij}], i, j \in \{1, .., M_i\}$ was calculated such that

$$\alpha_{ij} = \begin{cases} 1, & d(v_i, v_j) = 1 \\ 0, & \text{otherwise.} \end{cases}$$

A modified Dijkstra algorithm was used to calculate the lengths of the paths between the agents for each player [18]. Note that the lengths were computed only for the nodes from a given subset as described in Sec. II-B The length of the longest path was stored as the player's initial diameter. The code then checked if there was any competition for drones position by playing the first stage of the game. After a suitable number of game situations were generated and stored, the game itself was played. The game consisted of a compulsory first stage and then later stages run continuously until none of the players could decrease their diameter.

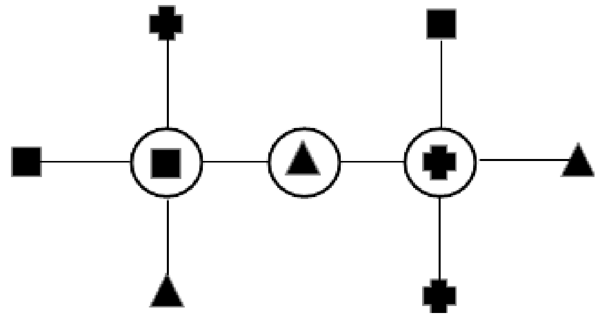

Figure 1. The figure illustrates the case of a 3-bridge connecting the agents of three different players (shown as a square, a triangle and a cross). The respective mobile agents are encircled. The figure presents a fragment of the total communication network.

We present two examples here, one is a game situation with one restricted area (lake), the second illustrates the situation when one of the players finds a new position for their drone which increases the diameter of another player. Players consider their own diameter optimisation, however sometimes the use of the second drone can be seen by us as a bridge building process. We should emphasize that the game is non-cooperative, and the players have no intention to build a bridge at any stage.

Example 1 which is depicted on Fig. 2 presents a situation when a bridge is not available at the first stage. The first player moves the drone to $(6, 4)$ (see Table I) and leaves the second player with 11 options minimising the diameters to 14 and 13 correspondingly, which means the pay-off after stage 1 is $[2, 2]$.

However, at stage 2 opportunities for the better use of

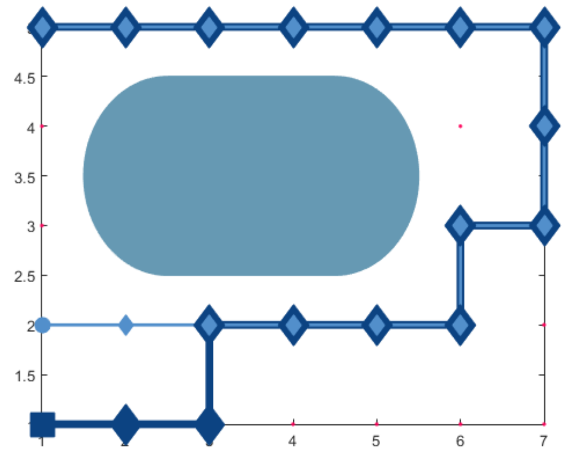

Figure 2. Initial placement of agents in Example 1. Two 2-bridges appeared $((3,3)-(4,5)$ and $(4,3)-(4,4))$ as results of the second stage of the game.

TABLE I
EXAMPLE 1: STAGE 1

Diameters initial	Drone 1 position	Drone 2 position	Diameters updated	Payoff current
$[16,15]$	$(6,4)$	$(7,2)$	$[14,13]$	$[2,2]$
$[16,15]$	$(6,4)$	$(1,3)$	$[14,13]$	$[2,2]$
$[16,15]$	$(6,4)$	$(2,3)$	$[14,13]$	$[2,2]$
$[16,15]$	$(6,4)$	$(3,3)$	$[14,13]$	$[2,2]$
$[16,15]$	$(6,4)$	$(4,3)$	$[14,13]$	$[2,2]$
$[16,15]$	$(6,4)$	$(5,3)$	$[14,13]$	$[2,2]$
$[16,15]$	$(6,4)$	$(1,4)$	$[14,13]$	$[2,2]$
$[16,15]$	$(6,4)$	$(2,4)$	$[14,13]$	$[2,2]$
$[16,15]$	$(6,4)$	$(3,4)$	$[14,13]$	$[2,2]$
$[16,15]$	$(6,4)$	$(4,4)$	$[14,13]$	$[2,2]$
$[16,15]$	$(6,4)$	$(5,4)$	$[14,13]$	$[2,2]$

the second player's drone for the first player are formed (see Table II), i.e., two 2-bridges appear $((3,3)-(4,5)$ and $(4,3)-(4,4))$. Each of the strategies result in pay-off $[16-10, 15-9] = [6,6]$. The third 2-bridge is also available $((5,3)-(5,4))$, but it only brings the payoff of $[16-12, 15-11] = [4,4]$. For the rest of the strategies, both players can not improve achievements of the first stage.

Example 2 has been generated randomly on the field with three lakes (Fig. 2) and it illustrates the non-cooperative nature of this game. The initial diameters for the players are 15 and 12 correspondingly (see Table III).

The first player has the only minimising position $(4,6)$ which brings 11 options for the second player. The pay-off function after stage 1 is $[15-13, 12-11] = [2,1]$ for all 11 strategies.

The next stage begins with the first player's attempt to minimise its diameter by using the second drone. Thus some of the strategies give multiple options and the total number of strategies becomes 13 (see Table IV). The first 5 strategies

TABLE II
EXAMPLE 1: STAGE 2

Drone 1 position	Drone 2 position	Diameters updated	Payoff total
$(6,4)$	$(7,2)$	$[14,13]$	$[2,2]$
$(6,4)$	$(1,3)$	$[14,13]$	$[2,2]$
$(6,4)$	$(2,3)$	$[14,13]$	$[2,2]$
$(6,4)\to(3,4)$	$(3,3)$	$[14\to10, 13\to9]$	$[6,6]$
$(6,4)\to(4,4)$	$(4,3)$	$[14\to10, 13\to9]$	$[6,6]$
$(6,4)\to(5,4)$	$(5,3)$	$[14\to12, 13\to11]$	$[4,4]$
$(6,4)$	$(1,4)$	$[14,13]$	$[2,2]$
$(6,4)$	$(2,4)$	$[14,13]$	$[2,2]$
$(6,4)\to(3,3)$	$(3,4)$	$[14\to10, 13\to9]$	$[6,6]$
$(6,4)\to(4,3)$	$(4,4)$	$[14\to10, 13\to9]$	$[6,6]$
$(6,4)\to(5,3)$	$(5,4)$	$[14\to12, 13\to11]$	$[4,4]$

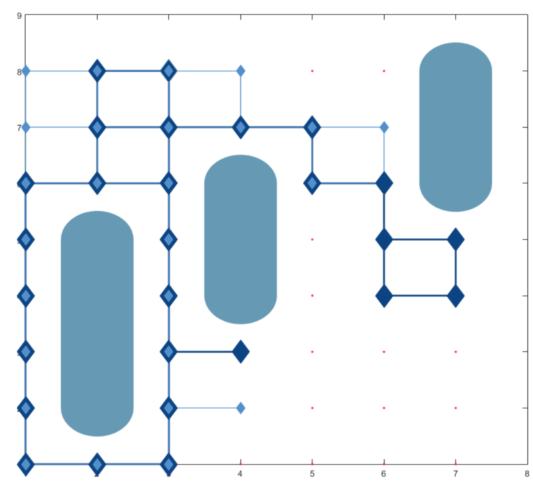

Figure 3. Initial placement of agents in Example 2. Only one 2-bridge position is potentially available $((4,5)-(5,5))$, however, the second player prefers to have a drone at $(4,6)$ in most of the cases.

TABLE III
EXAMPLE 2: STAGE 1

Diameters initial	Drone 1 position	Drone 2 position	Diameters updated	Payoff current
$[15,12]$	$(4,6)$	$(4,1)$	$[13,11]$	$[2,1]$
$[15,12]$	$(4,6)$	$(2,2)$	$[13,11]$	$[2,1]$
$[15,12]$	$(4,6)$	$(5,2)$	$[13,11]$	$[2,1]$
$[15,12]$	$(4,6)$	$(2,3)$	$[13,11]$	$[2,1]$
$[15,12]$	$(4,6)$	$(2,4)$	$[13,11]$	$[2,1]$
$[15,12]$	$(4,6)$	$(4,4)$	$[13,11]$	$[2,1]$
$[15,12]$	$(4,6)$	$(2,5)$	$[13,11]$	$[2,1]$
$[15,12]$	$(4,6)$	$(4,5)$	$[13,11]$	$[2,1]$
$[15,12]$	$(4,6)$	$(5,5)$	$[13,11]$	$[2,1]$
$[15,12]$	$(4,6)$	$(5,8)$	$[13,11]$	$[2,1]$
$[15,12]$	$(4,6)$	$(6,8)$	$[13,11]$	$[2,1]$

do not allow any improvements (as well as strategies $7, 10$ and 11) for both players, then make no changes to the drones' positions. However the $6-$th strategy (drone 1 at

TABLE IV
STAGE 2 EXAMPLE 2

Drone 1 position	Drone 2 position	Diameters updated	Payoff total
$(4,6)$	$(4,1)$	$[13,11]$	$[2,1]$
$(4,6)$	$(2,2)$	$[13,11]$	$[2,1]$
$(4,6)$	$(5,2)$	$[13,11]$	$[2,1]$
$(4,6)$	$(2,3)$	$[13,11]$	$[2,1]$
$(4,6)$	$(2,4)$	$[13,11]$	$[2,1]$
$(4,6) \to (5,4)$	$(4,4) \to (4,6)$	$[13 \to 12 \to 15, 11]$	$[0,1]$
$(4,6)$	$(2,5)$	$[13,11]$	$[2,1]$
$(4,6) \to (5,5)$	$(4,5)$	$[13 \to 12, 11]$	**[3,1]**
$(4,6) \to (5,5)$	$(4,5) \to (4,6)$	$[13 \to 12 \to 15, 11]$	$[0,1]$
$(4,6) \to (4,5)$	$(5,5)$	$[12,11]$	**[3,1]**
$(4,6) \to (4,5)$	$(5,5) \to (4,6)$	$[13 \to 12 \to 15, 11]$	$[0,1]$
$(4,6)$	$(5,8)$	$[13,11]$	$[2,1]$
$(4,6)$	$(6,8)$	$[13,11]$	$[2,1]$

$(4,6)$, drone 2 at $(4,4)$) gives an opportunity for the first player to use the second drone as a 2-bridge $((5,4),(4,4))$ which brings the diameter to 12 for both of the players. Now the second player finds a better position for the second drone, namely $(4,6)$, which bring the diameter down to 11, however that action increases the diameter of the first player to 15. Very similar situation happens with strategy 10 (see Table IV).

Strategy 8, where the first drone was at $(4,6)$ and the second one at $(4,5)$ after the first stage, allows the 2-bridge $((4,5)-(5,5))$ to appear after second stage minimisation.

Strategy number 10 is more special, it allows two options for the position of the second drone ($(4,5)$ and $(4,5)$) after the first drone was moved to $(5,5)$. Both makes the diameter of the second player being 11. However, only the first one leaves the diameter of the first player being 12, the second one makes it 15.

Thus two strategies lead to the bridge formation and bring the maximal total network improvement, but at the same time, two strategies degrade the total payoff comparing with the outcomes of the first stage.

V. CONCLUSION

A novel game-theoretic model of mobile agents placement on a mobile ad-hoc network was considered. It was shown that this non-cooperative problem have interesting properties. In particular, it may happen that the neither of the described strategies can lead the players to a Nash equilibrium solution. It is therefore interesting to consider different classes of strategies and solutions. The future research will include a detailed analysis of the observed phenomena and will concentrate on the design of new classes of strategies, including cooperative ones, to allow all players achieving the best possible outcomes.

ACKNOWLEDGEMENT

Dr Gromova, Dr Blakeway, and Dr Kirpichnikova have been partially supported by LMS Grant SC7-1415-12. Dr Gromova has been supported by the grant 9.38.245.2014 from St. Petersburg State University and Dr Gromov has been supported by the grant 9.50.1197.2014 from St. Petersburg State University.

REFERENCES

[1] N. Chaubey, A. Aggarwal, S. Gandhi, and K. A. Jani, "Performance analysis of TSDRP and AODV routing protocol under black hole attacks in manets by varying network size," in *Advanced Computing & Communication Technologies (ACCT), 2015 Fifth International Conference on*. IEEE, 2015, pp. 320–324.

[2] S. S. Anjum, R. M. Noor, and M. H. Anisi, "Survey on MANET based communication scenarios for search and rescue operations," in *IT Convergence and Security (ICITCS), 2015 5th International Conference on*. IEEE, 2015, pp. 1–5.

[3] H. Benkaouha, A. Abdelli, N. Badache, J. Ben-Othman, and L. Mokdad, "Towards improving failure detection in mobile ad hoc networks," in *2015 IEEE Global Communications Conference (GLOBECOM)*. IEEE, 2015, pp. 1–6.

[4] T. T. Son, H. Le Minh, G. Sexton, N. Aslam, and R. Boubezari, "A new mobility, energy and congestion aware routing scheme for MANETs," in *Communication Systems, Networks & Digital Signal Processing (CSNDSP), 2014 9th International Symposium on*. IEEE, 2014, pp. 771–775.

[5] S. Blakeway and A. Pullin, "The effect of node density on routing protocol performance in mobile ad-hoc networks," in *Proc. of the Convergence of Telecommunications, Networking and Broadcasting*, 2007.

[6] C. Xuelin and S. Zuxun, "An overview of slot assignment (SA) for TDMA," in *Signal Processing, Communications and Computing (ICSPCC), 2015 IEEE International Conference on*. IEEE, 2015, pp. 1–5.

[7] J. C. C. McKinsey, *Introduction to the Theory of Games*. Dover Publications, 2003.

[8] D. Fudenberg and J. Tirole, *Game Theory*. The MIT Press, 1991.

[9] T. Poongothai and K. Jayarajan, "A noncooperative game approach for intrusion detection in mobile adhoc networks," in *Computing, Communication and Networking, 2008. ICCCn 2008. International Conference on*. IEEE, 2008, pp. 1–4.

[10] F. Wang, Y. Mo, and B. Huang, "Defending reputation system against false recommendation in mobile ad hoc network," in *Networking, Sensing and Control, 2008. ICNSC 2008. IEEE International Conference on*. IEEE, 2008, pp. 488–493.

[11] F. Li, Y. Yang, and J. Wu, "Attack and flee: game-theory-based analysis on interactions among nodes in MANETs," *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on*, vol. 40, no. 3, pp. 612–622, 2010.

[12] X. Wang, R. Feng, Y. Wu, S. Che, and Y. Ren, "A game theoretic malicious nodes detection model in MANETs," in *Mobile Adhoc and Sensor Systems (MASS), 2012 IEEE 9th International Conference on*. IEEE, 2012, pp. 1–6.

[13] K. Wang, M. Wu, C. Ding, and W. Lu, "Game-based modeling of node cooperation in ad hoc networks," in *Wireless and Optical Communications Conference (WOCC), 2010 19th Annual*. IEEE, 2010, pp. 1–5.

[14] D. A. Novikov, "Games and networks," *Automation and Remote Control*, vol. 75, no. 6, pp. 1145–1154, 2014. [Online]. Available: http://dx.doi.org/10.1134/S0005117914060149

[15] Z. Han, D. Niyato, W. Saad, T. Başar, and A. Hjørungnes, *Game Theory in Wireless and Communication Networks. Theory, Models, and Applications*. Cambridge University Press, 2012.

[16] L. Petrosyan, N. Zenkevich, and E. Shevkoplyas, *Game Theory*. BHV-St. Petersburg, 2012, in Russian.

[17] H. W. Kuhn, "Extensive games and the problem of information," *Contributions to the Theory of Games*, vol. 2, no. 28, pp. 193–216, 1953.

[18] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to algorithms*, 2nd ed. MIT Press, 2001.